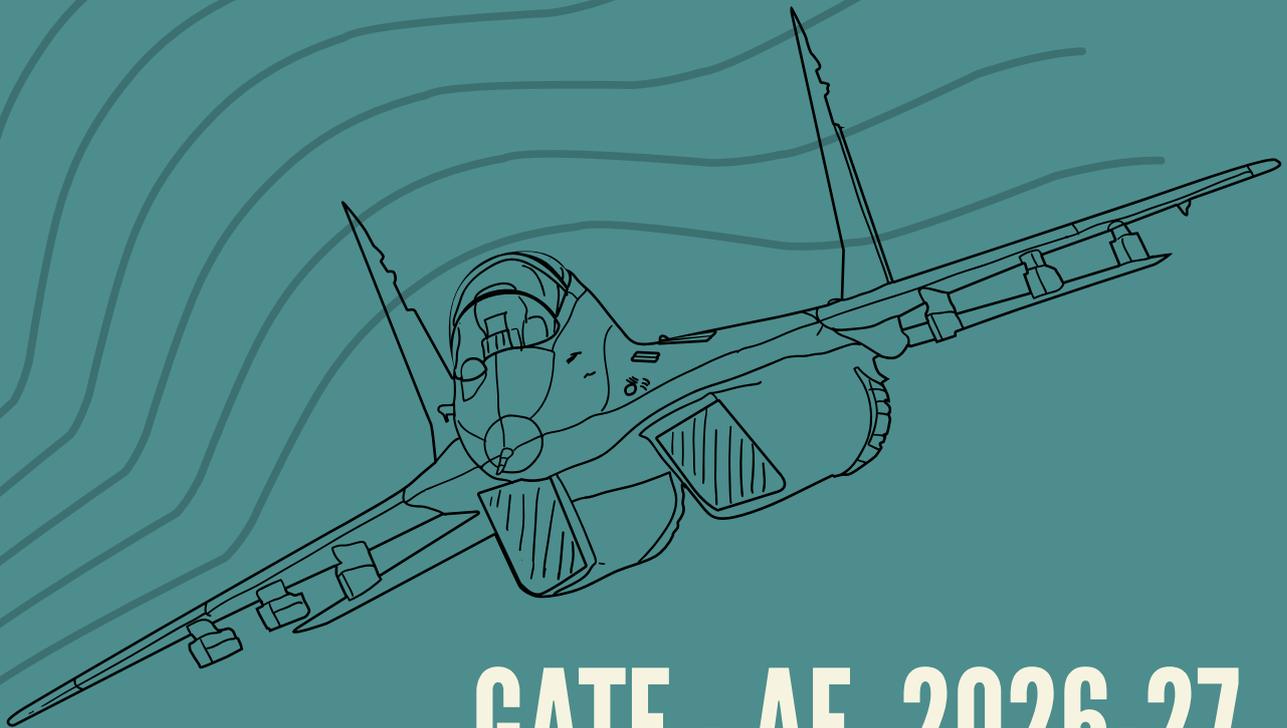


AERODYNAMICS

Edition 1



GATE - AE 2026-27

GATE AEROSPACE ENGINEERING

“The Ultimate Gift from IIT & IISc Graduates”
A Must-Have for Every GATE Aerospace Aspirant

Krishna Parmar

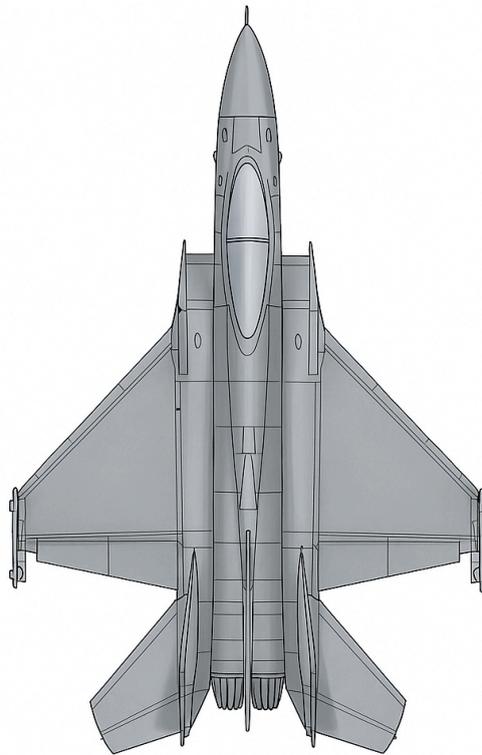
Aerodynamics

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A Must-Have for Every GATE Aerospace Aspirant

Edition 1



IITians Gate Academy

"Your Rank is Our Mission—IITians Gate Academy."



By Krishna Parmar

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Aerodynamics

Edition 1

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Preface

Preparing for the GATE Aerospace Engineering exam can be both exciting and challenging. With years of experience teaching and mentoring students, we at **IITians Gate Academy** recognized the need for a **comprehensive, exam-focused, and student-friendly set of books** to guide aspirants through this journey. This series, covering **Aerodynamics, Flight Mechanics, Aircraft Propulsion, and Aircraft Structures**, has been created with that goal in mind.

The **content and structure** of these books are designed to be **student-friendly**, making complex concepts easier to understand and apply. Every topic is aligned with the **latest GATE syllabus**, and chapters are organized to help aspirants build **strong conceptual foundations first, then master problem-solving techniques**.

What makes this series unique is the **expertise behind it**. The books have been authored by a team of **IIT and IISc graduates**, many of whom have **secured top 10 ranks in GATE**. Their combined experience in **teaching, exam strategy, and real-world aerospace knowledge** ensures that the material is not only accurate but also highly effective for exam preparation.

We have included **hand-drawn diagrams, solved examples, and practice questions** to provide clarity, reinforce learning, and give students the confidence to tackle even the toughest GATE problems. These books are intended to be more than textbooks—they are **friends and mentors** to aspirants on their path to success.

It is our sincere hope that this series helps students achieve their **dream rank and goals in GATE Aerospace Engineering**, and inspires them to explore the vast and fascinating field of aerospace with confidence and curiosity.

IITians Gate Academy

"Your Rank is Our Mission – IITians Gate Academy."



Praveen Chaudhary

IAS Officer | M.Tech IIT Madras | Aerospace Engineer
Turned Civil Servant

Foreword

It gives me immense pleasure to write this foreword for my dear friend and mentor, Krishna Parmar. Our journey together began at Madras, where we shared not just classrooms but a deep passion for learning and growth. During our GATE preparation, Krishna's mentorship had a major influence on my understanding and approach to the exam.

As my classmate and guide, he had a rare ability to simplify complex concepts with clarity and precision. His structured approach, disciplined preparation style, and passionate commitment helped me strengthen my fundamentals across multiple subjects. Under his guidance, I learned the importance of consistency, rigorous practice, and self-belief—values that continue to guide me long after our student days.

The hours we spent discussing problems, analyzing mock tests, and reviewing previous year papers were invaluable. Krishna's encouragement and insight not only boosted my confidence but also transformed the way I approached challenges, both academic and personal.

I owe a great deal of my success in the GATE examination to his guidance and friendship.

This book reflects the same qualities that defined him as a mentor - clarity, depth, and a genuine desire to help others excel. I am certain that readers will find in these pages both knowledge and inspiration.

Praveen Chaudhary

Introduction

Welcome to the GATE Aerospace Engineering series by **IITians Gate Academy** – where **“Your Rank is Our Mission”**. These books have been carefully crafted for serious aspirants aiming to excel in the GATE Aerospace exam, covering **Aerodynamics, Flight Mechanics, Aircraft Propulsion, and Aircraft Structures**.

At IITians Gate Academy, we believe that success in competitive exams comes not just from hard work, but from **smart, focused preparation**. Our team consists of IIT and IISc graduates who have not only cleared GATE themselves, many securing **top 10 ranks**, but also mastered the art of **teaching complex concepts in a simple, precise, and exam-oriented manner**.

Each book in this series is designed with the GATE aspirant in mind. The content is **concise, to-the-point, and curated exclusively for exam preparation**. Concepts are explained with **clarity and brevity**, reinforced with **hand-drawn diagrams, solved examples, and practice questions** that reflect the latest GATE trends. Our approach ensures that you **learn faster, retain better, and solve questions with confidence**.

What makes these books unique is our **exclusive study material** – created from scratch by our expert team, based on years of research, teaching experience, and **hands-on problem-solving**. Every chapter is structured to help you **build conceptual understanding first, then master application**, so you are fully prepared to tackle even the toughest questions.

By choosing IITians Gate Academy’s books, you are not just studying from a textbook – you are getting the **experience, guidance, and mentorship of top-ranked GATE graduates**, distilled into materials that are easy to understand and highly effective.

Whether you are revising for quick recall, solving practice problems, or preparing for advanced concepts, these books are your **all-in-one resource** to achieve your GATE aspirations.

- IITians Gate Academy

In this book, we have covered the following Aerodynamics syllabus, focusing on both fundamental and advanced concepts. The topics are carefully structured to strengthen the understanding required for academic learning as well as competitive exams like GATE.

Aerodynamics

Core Topics:

Basic Fluid Mechanics: Conservation laws: Mass, momentum and energy (Integral and differential form); Dimensional analysis and dynamic similarity;

Potential flow theory: sources, sinks, doublets, line vortex and their superposition. Elementary ideas of viscous flows including boundary layers.

Airfoils and wings: Airfoil nomenclature; Aerodynamic coefficients: lift, drag and moment; Kutta Joukowski theorem; Thin airfoil theory, Kutta condition, starting vortex; Finite wing theory: Induced drag, Prandtl lifting line theory; Critical and drag divergence Mach number.

Compressible Flows: Basic concepts of compressibility, One-dimensional compressible flows, Isentropic flows, Fanno flow, Rayleigh flow; Normal and oblique shocks, Prandtl-Meyer flow; Flow through nozzles and diffusers.

Special Topics: Wind Tunnel Testing: Measurement and visualization techniques. Shock - boundary layer interaction.

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Basic Fluid Mechanics

Chapter 1

1.1 Introduction—Basic Fluid Mechanics

Fluid mechanics studies the behavior of fluids (liquids and gases) at rest and in motion. It is based on fundamental physical principles such as mass conservation, Newton's second law, and energy conservation. To analyze fluid flow, we use different fluid models—finite control volume, infinitesimal fluid element, and molecular approach—which help simplify complex flow problems. Understanding these models and principles provides the foundation for solving a wide range of aerodynamic and engineering applications.

Before we begin analyzing aerodynamic problems, it is important to understand the different types of fluid flow. Knowing the types of flow helps us classify and predict fluid behavior, which is essential for studying aerodynamics, designing aircraft, and solving engineering problems effectively.

1.2 Types of Flow

1.2.1 Continuum Versus Free Molecule Flow

Consider flow over a body, say a circular cylinder of diameter d . The gas is made of molecules moving randomly. The average distance a molecule travels before colliding with another is called the **mean free path** (λ).

- If $\lambda \ll d$, the molecules impact the body surface so frequently that the body cannot distinguish the individual molecular collisions, and the surface feels the fluid as a continuous medium; the fluid behaves like a **continuous medium**. This is called **continuum flow**.
- If $\lambda \approx d$, molecules are far apart, and collisions with the body surface occur only infrequently, and the body surface can distinctly feel each molecular impact, and this is called **free molecular flow**.

In reality:

- Spacecraft (like the space shuttle) experience **free molecular flow** at the outer edge of the atmosphere, where density is very low.
- Most practical aerodynamics (airplanes, cars, turbines) involve **continuum flow**.

- **Low-density flows** lie between the two extremes but are less common.

In aerodynamics, we mainly focus on **continuum flow**.

1.2.2 Inviscid Versus Viscous Flow

- **Viscous flow:** All real flows have transport phenomena—mass diffusion, viscosity (friction), and thermal conduction. These effects make the flow viscous.
- **Inviscid flow:** If we neglect friction, conduction, and diffusion, the flow is called inviscid. Inviscid flows don't truly exist in nature, but many high Reynolds number flows can be modeled as inviscid.

Viscous Flow (Viscosity $\mu \neq 0$)

$$\tau = \textit{shear stress}$$

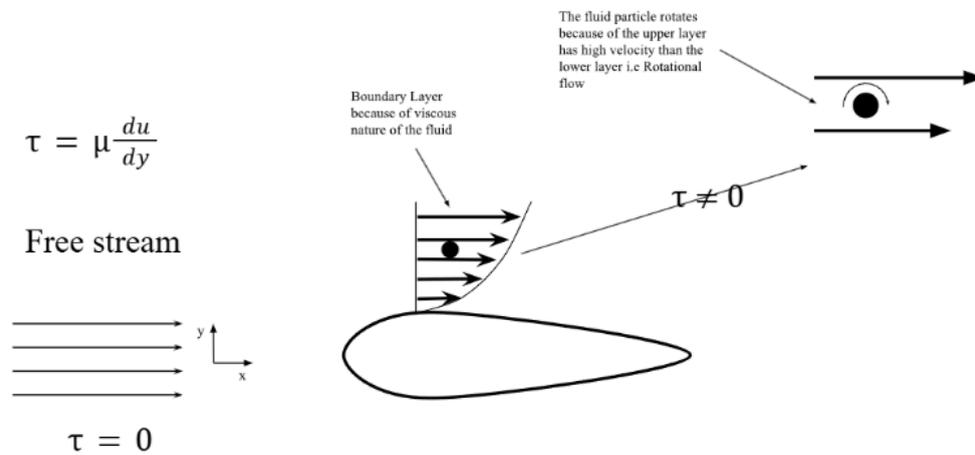


Fig. 1.2 a

Ideal Flow (Viscosity $\mu = 0$)

$$\tau = \textit{shear stress}$$

$$\tau = \mu \frac{du}{dy}$$

$$\tau = 0$$



Fig. 1.2 b

- In such cases:
 - Only a thin boundary layer near the body surface is viscous.
 - The outer flow can be treated as inviscid.

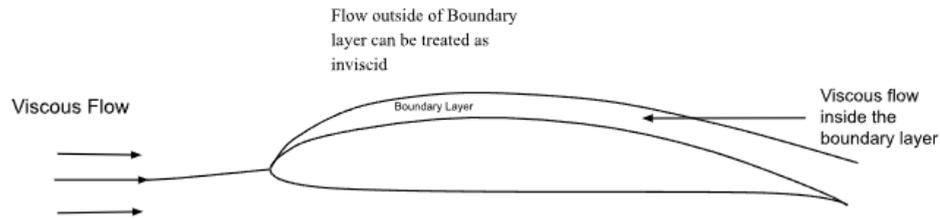


Fig. 1.2 c

- Inviscid theory predicts pressure distribution, lift, and streamlines well but cannot predict drag (since drag comes mainly from friction).

So, in aerodynamics we

- Use **inviscid theory** for pressure and lift (outside the boundary layer). ★
- Use **viscous flow analysis** for drag, separation, and wakes. ★

1.2.3 Incompressible vs Compressible Flows

Incompressible Flow

- Density (ρ) is constant.
- In reality, *perfectly* incompressible flow does not exist in nature.
- Used as an **assumption** in many cases (to simplify problems).
- Examples:
 - Liquids (like water) are usually treated as incompressible.
 - For gases at low Mach number ($M < 0.3$) or $V \leq 100$ m/s, the density change is so small that we can assume $\rho = \text{Constant}$.

Compressible Flow

- Density (ρ) changes with pressure, temperature, and velocity.
- Must be considered at **high speeds** ($M > 0.3$).
- Important in modern high-speed and supersonic aircraft, missiles, and rockets.

1.2.4 Laminar vs. Turbulent Flow

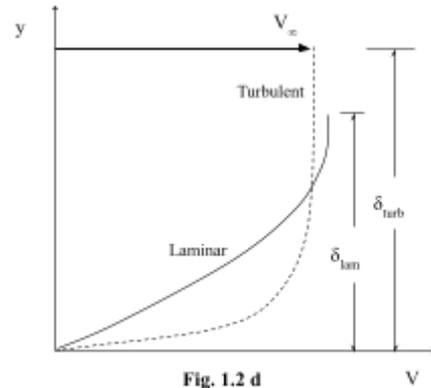
Laminar Flow

- Streamlines are smooth and regular.

- A fluid element moves in an orderly manner along a streamline.
- The velocity profile in the boundary layer decreases gradually from free-stream to zero at the wall.
- Low shear stress → skin friction drag is smaller.
- Lower aerodynamic heating compared to turbulent flow.

Turbulent Flow

- Streamlines are irregular and chaotic.
- Fluid motion is random, mixed-up, and fluctuating.
- The velocity profile is fuller (stays close to the free stream until near the wall, then drops rapidly).
- High shear stress → skin friction drag is larger.
- Higher aerodynamic heating, especially critical at hypersonic speeds (can be up to 10× greater than laminar).



1.2.5 Steady Flow vs Unsteady Flow

Steady Flow

Definition: Flow properties (velocity, pressure, density, and temperature) at a point do not change with time.

Mathematically: $\frac{\partial(\text{fluid property})}{\partial t} = 0$

Example:

- Water flowing at a constant rate through a garden hose.
- Airflow over an aircraft cruising at constant altitude and speed.

Unsteady Flow

Definition: Flow properties at a point change with time.

Mathematically: $\frac{\partial(\text{fluid property})}{\partial t} \neq 0$

Example:

- Water flows when you turn a tap on/off.

- Airflow over an aircraft during takeoff, gusts, or maneuvers.

Note: Look section 6.10, “Important Conclusion” chapter, for a very important example about steady and Unsteady flow

1.2.6 Uniform Flow Vs Non-Uniform Flow

Uniform Flow

In uniform flow, the fluid properties (such as velocity, pressure, and density) are the same at every point in the flow at a given time, i.e., no change from point to point along the flow.

Example: Water flowing at a constant speed in a wide, straight river.

Non-Uniform Flow

In non-uniform flow, the fluid properties vary from point to point in the flow at a given time, i.e., fluid behavior changes depending on the location in the flow.

Example: Water flowing in a river that narrows or widens, causing speed to change.

1.2.7 Irrotational Flow Vs Rotational Flow

Irrotational Flow

In irrotational flow, the fluid particles do not spin or rotate about their own axes as they move. The motion of the fluid is smooth, and there is no angular velocity at any point, i.e., fluid elements move without rotation.

Example: Flow of air over a streamlined wing where the layers slide past each other without swirling.

In irrotational flow, total pressure remains same throughout the flow field. ★

➔ Velocity potential function ϕ particularly defined for study of irrotational flow only. ★

Rotational Flow

In rotational flow, the fluid particles rotate about their own axes as they move along the flow. This creates vortices or swirling motion in the fluid, i.e., fluid elements spin while moving

Example: Water swirling near a drain or in a tornado.

In Rotational flow total pressure varies from streamline to streamline but remains same along a single streamline. ★

➤ The Earth is both revolving around the Sun and rotating about its own axis. If we think of the Earth as a fluid particle, this motion is a perfect example of rotational flow, because the particle is spinning while moving.

➤ However, if the Earth only revolved around the Sun without rotating on its axis, then its motion would be a perfect example of irrotational flow, since there would be no rotation of the particle itself.

Hello Guyz,

Always remember—Nature follows three basic fundamental laws:

1. Mass is conserved
2. Momentum is conserved
3. Energy is conserved

Whenever we want to solve a real-world problem, we use these three fundamental laws, applying them according to the situation. The most important part is to apply them correctly and in the simplest way possible, based on the physics of the problem. In aerodynamics, the three fundamental equations—conservation of mass (continuity), momentum, and energy—are applied to analyze real-world flow problems. These equations can be applied at different levels depending on the scale and nature of the analysis:

1. **Finite Control Volume:** A selected region in space through which fluid flows or moves with flow with a fixed mass of fluid inside it. The governing equations are applied to the entire control volume to calculate fluxes of mass, momentum, and energy across its boundaries.
2. **Infinitesimal Fluid Element:** By considering a small fluid element (differential volume), the fundamental equations can be expressed in differential form, giving local values of velocity, pressure, and temperature at each point in the flow field.
3. **Molecular Approach:** At the microscopic level, fluid behavior is analyzed by tracking individual molecules, considering molecular collisions and statistical behavior, which leads to the derivation of macroscopic properties like density, viscosity, and thermal conductivity.

The key is to **choose the appropriate approach** based on the physics of the problem. Applying these equations correctly—whether over a finite control volume, a differential fluid

element, or using molecular theory—ensures accurate and practical solutions for aerodynamic analysis.

Important Points:

- The continuity equation is a mathematical statement of the fundamental principle that mass is conserved.
- The momentum equation is a mathematical statement of Newton's second law.
- The energy equation is a mathematical statement of energy conservation, i.e., the first law of thermodynamics.

For example, let's see how we can use the energy conservation law to solve problems in aerodynamics. Now we will develop an energy equation that suits our aerodynamics. So we apply this energy equation to solve the aerodynamic problems.

1.3 Energy Conservation Law $[E_{in} - E_{out} = \Delta E]$

Energy can't be created nor be destroyed; it can only transfer from one form to another form.

So we can say mathematically $E_{in} - E_{out} = \Delta E$ -----eq 1

(Fundamental equation of the law of energy conservation)

Suppose a stationary, closed system takes Q heat and delivers W work, and its change in internal energy is dU . (Can think about the piston-cylinder arrangement in a IC engine, which you have read in thermodynamics.)

Note:

1. **Closed system:** In which energy transfer can take place across the boundary of the system.
2. **Open System:** energy and mass transfer can both take place across the boundary of the system.

→ Always remember that energy has two main forms: **heat & work**. All energy coming into the system is positive, and all forms of energy leaving the system are considered negative.

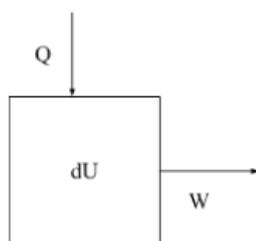


Fig. 1.3 a

So we can write

Equation 1 as,

$$Q - W = dU \text{ -----eq 2}$$

Closed and stationary,

$$\text{Where, } E_{in} = Q, E_{out} = W$$

Potential Flow Theory

Chapter 2

Inviscid and Incompressible Flow

2.1 Fundamentals of Inviscid and Incompressible Flow

An inviscid, incompressible fluid is sometimes called an ideal or perfect fluid. However, these terms can cause confusion with ideal gases or perfect gases from thermodynamics. To avoid this, it is better to use the precise term “**inviscid, incompressible flow**” instead of “ideal” or “perfect fluid.”

The study of inviscid, incompressible fluid focuses on three main areas: the development of Bernoulli’s equation with some simple applications, an introduction to Laplace’s equation as the governing equation for inviscid, incompressible, and irrotational flow, and a study of elementary flow patterns. These flow patterns show how non-lifting and lifting flows over a circular cylinder can be modeled.

We know that

$$dp = -\rho V dV \quad \text{-----Eq 1}$$

Equation 1 is **Euler’s equation, which** applies to **inviscid flow** with no body forces. It relates the change in velocity dV along a streamline to the change in pressure dp along the same streamline. ★★

For the special case of **incompressible flow** $\rho = \text{constant}$, Euler’s equation can be directly integrated between any two points (1 and 2) along a streamline. This integration leads to a very important result: **Bernoulli’s equation**.

$$\begin{aligned} \Rightarrow \int_{p_1}^{p_2} dp &= -\rho \int_{V_1}^{V_2} V dV \\ p_2 - p_1 &= -\rho \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) \end{aligned}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{-----Eq2}$$

Equation (2) represents Bernoulli’s equation. It shows the relationship between the **pressure and velocity** at location 1 on a streamline and the **pressure and velocity** at another location 2 along the same streamline. Equation 2 can also be written

$$p + \frac{1}{2}\rho V^2 = \text{Constant}$$

➔ Along a Streamline

If the flow **irrotational** then

$$p + \frac{1}{2}\rho V^2 = \text{Constant}$$

➔ Throughout the flow

Bernoulli's equation comes from the momentum equation, so it is essentially a statement of Newton's second law for an inviscid, incompressible flow without body forces. The terms in Bernoulli's equation represent energy per unit volume—for example, $\frac{1}{2}\rho V^2$ is the *kinetic energy per unit volume*. In this sense, Bernoulli's equation also expresses the *mechanical energy balance: the work done by pressure forces on the fluid equals the change in its kinetic energy*.

Bernoulli's equation can also be derived from the general energy equation, though that derivation is optional. The fact that it can be viewed either as a momentum or an energy relation shows that for inviscid, incompressible flows, the continuity and momentum equations are enough—the energy equation is not strictly needed.

Most problems related to an inviscid, incompressible flow are solved as follows:

1. First, determine the **velocity field** using the governing equations for inviscid, incompressible flow.
2. Then, calculate the **pressure field** from Bernoulli's equation.

Example: Consider the inviscid, incompressible flow of air along a streamline. The air density along the streamline is 1.20 kg/m^3 , which is standard atmospheric density at sea level. At point 1 on the streamline, the pressure and velocity are 101.3 kPa and 3.05 m/s , respectively. Further downstream, at point 2 on the streamline, the velocity is 57.9 m/s .

1. Calculate the pressure at point 2.
2. Comment on the relative change in pressure compared to the change in velocity along the streamline.

Solution: Using Bernoulli's equation for incompressible, inviscid flow:

$$\begin{aligned} p_1 + \frac{1}{2}\rho V_1^2 &= p_2 + \frac{1}{2}\rho V_2^2 \\ p_2 &= p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2) \\ &= 101300 + \frac{1}{2} \times 1.20 \times (3.05^2 - 57.9^2) = 99.3 \text{ kPa} \end{aligned}$$

Conclusion: Velocity increases from 3.05 m/s to 57.9 m/s, but pressure decreases slightly from 101.3 kPa to 99.3 kPa.

i.e., in low-speed flows, only a small pressure change is needed to produce a large velocity change—this explains why small barometric differences can create strong winds.

2.2 Substantial Derivatives (Total Derivatives)

(Time rate of change following a moving fluid element)

Consider a **small fluid element** moving through a **flow field**, as shown in the figure below. The **velocity field** of the fluid is given by:

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

where u , v and w are the velocity components along the x , y and z directions, respectively.

Consider a unsteady flow where u , v & w are function of both space & time

$$u = u(x, y, z, t)$$

$$v = v(x, y, z, t)$$

$$w = w(x, y, z, t)$$

The scalar density field is given by,

$$\rho = \rho(x, y, z, t)$$

At time t_1 , fluid element is located at point 1

$$\rho_1 = \rho(x_1, y_1, z_1, t_1)$$

At time $t = t_2$ point 2,

$$\rho_2 = \rho(x_2, y_2, z_2, t_2)$$

Now, Since $\rho = \rho(x, y, z, t)$

We can expand this function using the Taylor series.

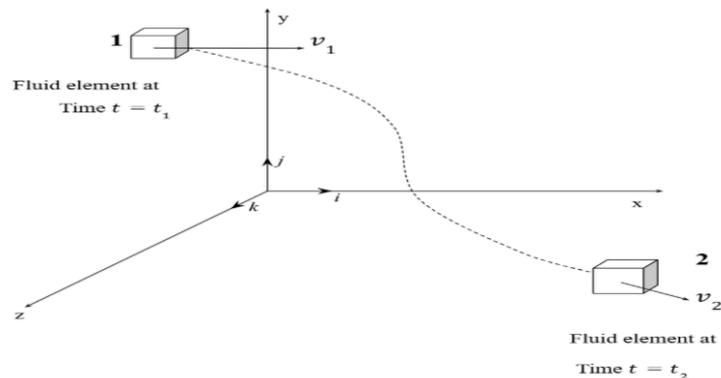


Fig. 2.2 a

$$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1(x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1(y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1(z_2 - z_1) + \left(\frac{\partial \rho}{\partial t}\right)_1(t_2 - t_1) + \text{higher-order terms}$$

Divided by $(t_2 - t_1)$

$$\frac{\rho_2 - \rho_1}{t_2 - t_1} = \left(\frac{\partial \rho}{\partial x}\right)_1 \left(\frac{x_2 - x_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial y}\right)_1 \left(\frac{y_2 - y_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial z}\right)_1 \left(\frac{z_2 - z_1}{t_2 - t_1}\right) + \left(\frac{\partial \rho}{\partial t}\right)_1 \text{-----Eq 1}$$

$\frac{\rho_2 - \rho_1}{t_2 - t_1}$ The term is physically the average time rate of change in the density of the fluid element as it moves from point 1 to point 2.

$$\Rightarrow \lim_{t_2 \rightarrow t_1} \frac{\rho_2 - \rho_1}{t_2 - t_1} = \frac{D\rho}{Dt}$$

$\frac{D\rho}{Dt}$ → is the time rate of change of density of a given fluid element as it moves through space.

This $\frac{D\rho}{Dt}$ is different from $\left(\frac{\partial \rho}{\partial t}\right)_1$,

- $\left(\frac{\partial \rho}{\partial t}\right)_1$, which is the physical time rate of change in density at fixed point 1.
- We fixed our eyes on the stationary point 1 and watched the density change due to transient fluctuation in the flow field.

Thus, they $\frac{D\rho}{Dt}$ & $\frac{\partial \rho}{\partial t}$ are physically & numerically different quantities.

We know that,

$$\lim_{t_2 \rightarrow t_1} \frac{x_2 - x_1}{t_2 - t_1} = u$$

$$\lim_{t_2 \rightarrow t_1} \frac{y_2 - y_1}{t_2 - t_1} = v$$

$$\lim_{t_2 \rightarrow t_1} \frac{z_2 - z_1}{t_2 - t_1} = w$$

We can write equation 1 when $t_2 \rightarrow t_1$

$$\frac{D\rho}{Dt} = u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t}$$

Now, we can obtain an expression for the substantial derivatives in Cartesian coordinates.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

$$\therefore \nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (V \cdot \nabla)$$

$\frac{D}{Dt}$ → **called substantial or total derivative**

→ Which is physically the time rate of change following a moving fluid element.

$\frac{\partial}{\partial t}$ → **local derivative**

→ Which is physically the time rate of change at a fixed point.

$V \cdot \nabla$ → **is called a consecutive or spatial derivative** (change with space or distance).

→ Which is physically the time rate of change due to the movement of the fluid element from one location to another in the flow field, where flow properties are spatially different.

2.3 Fundamental Equations in terms of the Substantial Derivative

Now we will express the continuity, momentum, and energy equations in terms of the substantial derivatives.

We know the vector identity:

$$\nabla \cdot (\rho V) = \rho \nabla \cdot V + V \cdot \nabla \rho$$

This identity states that the divergence of a scalar times a vector is equal to the scalar times the divergence of the vector plus the dot product of the vector and the gradient of the scalar.

1. First, consider the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0$$

By using the above vector identity, we can write the continuity equation

$$\frac{\partial \rho}{\partial t} + V \cdot \nabla \rho + \rho \nabla \cdot V = 0$$

We know the substantial derivatives;

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (V \cdot \nabla) \Rightarrow \frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + V \cdot \nabla \rho$$

by using this, we can write the continuity equation

$$\frac{D\rho}{Dt} + \rho \nabla \cdot V = 0$$

This formula is the form of the **continuity equation written in terms of the substantial derivative**.

2. Momentum Equation

The x-component of the momentum equation

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u V) = -\frac{\partial p}{\partial x} + \rho f_x + (F_x)_{viscous}$$

The first term can be written as $\frac{\partial(\rho u)}{\partial t} = \rho \frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial t}$ and for the second term, we can use the vector identity (treat u as a scalar and ρV as a vector). We will get the x-component of the **momentum equation in terms of the substantial derivative**.

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + (F_x)_{viscous}$$

Similarly, for y & z-component

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y + (F_y)_{viscous}$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z + (F_z)_{viscous}$$

3. Energy Equation: In the same way, we can write the **energy equation in terms of the substantial derivative**.

$$\rho \frac{D(e + \frac{v^2}{2})}{Dt} = \rho \dot{q} - \nabla \cdot (pV) + \rho(f \cdot V) + \dot{Q}_{viscous} + W_{viscous}$$

	<p>The conservation form of the fundamental equations. Sometimes these equations are called the divergence form (as the divergence terms are on the left side).</p>	<p>These equations with the substantial derivative on the left side, are called the nonconservation form</p>
<p>Continuity Equation</p>	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0$	$\frac{D\rho}{Dt} + \rho \nabla \cdot V = 0$

<p>Momentum Equations</p>	$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u V) = -\frac{\partial p}{\partial x} + \rho f_x + (F_x)_{viscous}$ $\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho v V) = -\frac{\partial p}{\partial y} + \rho f_y + (F_y)_{viscous}$ $\frac{\partial(\rho w)}{\partial t} + \nabla \cdot (\rho w V) = -\frac{\partial p}{\partial z} + \rho f_z + (F_z)_{viscous}$	$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + (F_x)_{viscous}$ $\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \rho f_y + (F_y)_{viscous}$ $\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \rho f_z + (F_z)_{viscous}$
<p>Energy Equation</p>	$\frac{\partial}{\partial t} \left[\rho \left(e + \frac{v^2}{2} \right) \right] + \nabla \cdot \left[\rho \left(e + \frac{v^2}{2} \right) V \right]$ $= \rho \dot{q} - \nabla \cdot (pV) + \rho (f \cdot V) + \dot{Q}_{viscous} + W_{viscous}$	$\rho \frac{D(e + \frac{v^2}{2})}{Dt}$ $= \rho \dot{q} - \nabla \cdot (pV) + \rho (f \cdot V) + \dot{Q}_{viscous} + W_{viscous}$

Note: Both forms are equally valid statements of the fundamental principles

2.4 Angular Velocity, Vorticity, And Strain

We have often used the idea of a fluid element moving in the flow field. Here, we look more closely at its motion, focusing on how its orientation and shape change when it moves along a streamline. In this study, we introduce vorticity, a very crucial concept in theoretical aerodynamics.

An infinitesimal fluid element moving in a flow field not only translates along a streamline but may also *rotate* and *distort* in shape. The

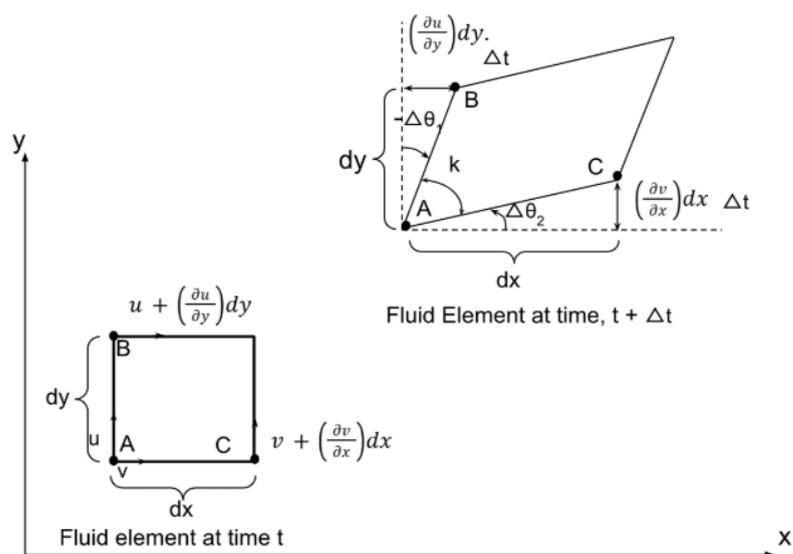


Figure 1] Rotation and distortion of a fluid element.

Fig. 2.4 a

Airfoils and Wings

Chapter 3

3.1 Airfoil and its Nomenclature

"Airfoil" (American English) or "aerofoil" (British English) nomenclature refers to the terminology and key geometric features used to describe the shape and characteristics of an aircraft wing section.

- It is the **cross-section** of aircraft wing
- This structure is commonly referred to as the **2-D wing**.
- Produces much **more lift than drag**.
- The aerofoil shape is designed to ensure **streamline flow** as far as possible.
- The **leading edge is rounded** to ensure **smooth flow**.
- The **trailing edge is sharp** so that the **Kutta condition** may be satisfied, the **wake region kept thin**, and the region of **separation kept as small** as possible.

1. Leading Edge—The most forward point of the airfoil where airflow encounters the surface first.

2. Trailing Edge—The rearmost point of the airfoil where the upper and lower surfaces meet and airflow leaves the airfoil.

3. Chord Line—This is a reference straight line that is drawn through the centers of the curvature of the leading and trailing edges.

In simpler terms, it is a straight line that connects the leading edge to the trailing edge.

4. Chord Length (c)—which is the distance between the points of intersection of the chord line with the leading and trailing edges.

The chord line measures the distance between the leading and trailing edges.

5. Camber Line—This is the locus of points halfway between the upper and lower surfaces, distances being measured normal to the camber line.

6. Camber—which is the distance, d , of the camber line from the chord line.

- It is expressed as a fraction of the chord, i.e., d/c .
- The camber is considered positive when the camber line is above the chord line, indicating that the upper surface is more curved than the lower surface.
- Negative if the camber line lies below the chord line (lower surface more curved than the upper).

Aerodynamics

- It is zero camber if the section is symmetrical about the chord line (i.e., the camber line lies on the chord line)

7. Thickness—The thickness is measured normal to the chord line. It is usually expressed as a fraction of the chord, t/c , called the thickness/chord ratio

- The distance between the upper and lower surfaces typically varies along the chord.

8. Maximum Thickness

- The greatest distance between the upper and lower surfaces is often expressed as a percentage of the chord.

9. Maximum Camber

- The maximum distance between the mean camber line and the chord line.

10. Angle of Attack (α)

- The angle between the chord line and the relative free stream airflow.

11. Aerodynamic Center

- The Aerodynamic Center is the point along the chord line, typically around 25% of the chord for subsonic airfoils, where the pitching moment remains constant regardless of the angle of attack.

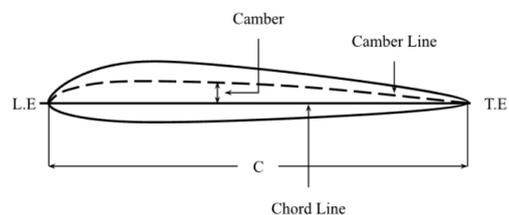


Fig. 3.1 a

12. Center of Pressure

- The Center of Pressure is the point along the chord line where the total sum of aerodynamic forces acts, and around which the moment is always zero.

13. Leading Edge Radius

- The radius of curvature of the leading edge influences flow separation and stall behavior.

3.2 Geometric Construction of an Airfoil Section

1. Set up the leading and trailing edges and join them to construct a chord line.

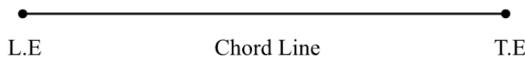


Fig. 3.2 a

2. Add curvature with a camber line.

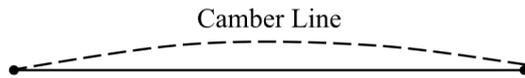


Fig. 3.2 b

3. Draw the upper and lower surfaces, with equal distances at each point from the camber line.

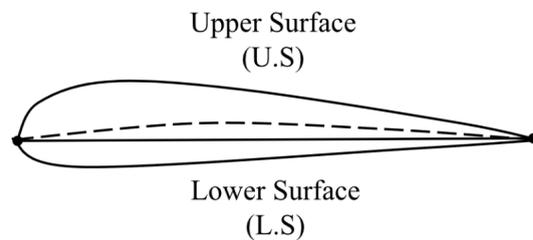


Fig. 3.2 c

4. Final airfoil shape.



Fig. 3.2 d

3.3 Important Points about the Airfoil Section

- The airfoil shape is designed to promote streamlined flow; the leading edge is rounded for smooth airflow, while the trailing edge is sharp to satisfy the Kutta condition, ensuring a thin wake region and minimal flow separation.
- This feature helps achieve high lift and low drag.

3.3.1 Symmetric Airfoil

If you look at the pressure distribution around a symmetric airfoil at zero AoA, there is equal acceleration of the flow around the upper & lower surfaces as the flow

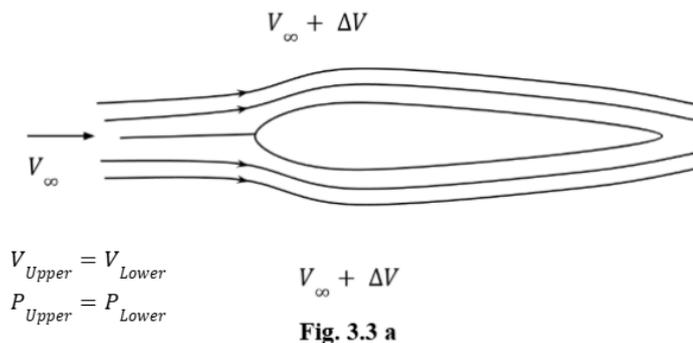


Fig. 3.3 a

encounters the same curvature on both sides of the airfoil.

→ So there is equal velocity on the upper and lower surfaces, i.e., equal pressure. It means there is no pressure difference. So, there is no net force.

→ Now if you increase the AoA, α greater than zero, then there is more acceleration at the upper surface than the lower surface, because the flow encounters more curvature at the upper surface, so there is more velocity at the upper surface, i.e., much less pressure than the lower surface, i.e., a net force exists, i.e., lift.

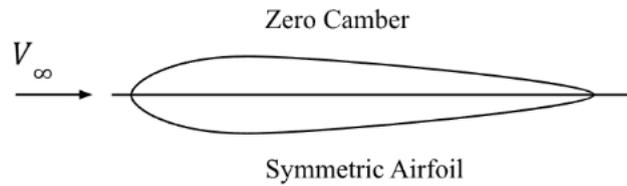


Fig. 3.3 b

→ If AoA increases further than the lift increase, if you further increase the AoA more than the stalling AoA, then stall happens and lift decreases drastically. We can discuss that later.

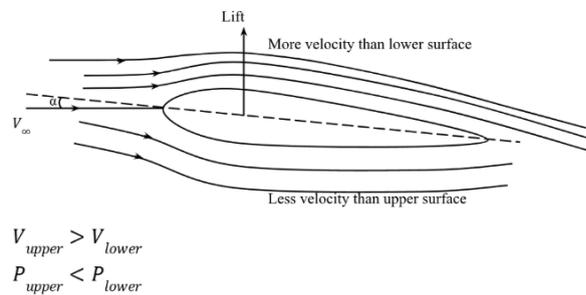


Fig. 3.3 c

→ Camber line = chord line.

→ No lift at $\alpha = 0$

→ The angle of zero lift is also zero

i. e $\alpha_{L=0} = 0^\circ$

$\alpha \rightarrow$ is the angle of attack

This refers to the angle that exists between the free airstream and the chord line.

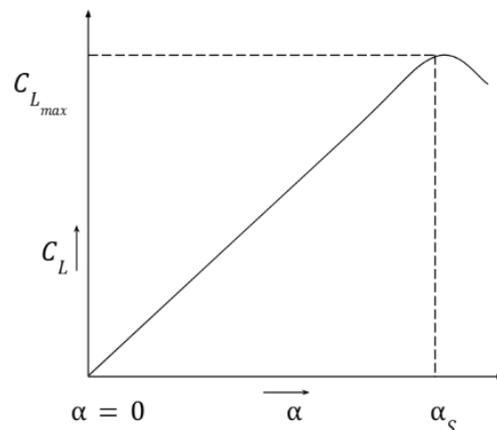


Fig. 3.3 d

3.3.2 Unsymmetric Airfoil

Positive Cambered Airfoil

For a positively cambered airfoil, lift occurs at $\alpha = 0$ because the unsymmetric design causes the flow to encounter more curvature on the upper surface than on the lower surface at zero-degree angle of attack.

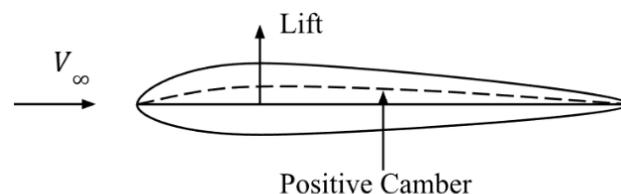


Fig. 3.3 e

→ The camber line is above the chord line.

→ Positive lift at $\alpha = 0^\circ$

i. e $\alpha_{L=0} < 0^\circ$

In other words, the chord line must be angled negatively in relation to the free stream to achieve zero lift. *i. e* $\alpha_{L=0} < 0^\circ$

→ To achieve zero lift, the airfoil should be oriented in a nose-down attitude such that the airflow encounters equal curvature on both the upper and lower surfaces. In this setup, the airflow speed is the same on both surfaces, so there's no pressure difference.

Consequently, no lift is generated.

→ For a positively cambered airfoil at the zero-lift condition, the pressure distribution around the airfoil generates a nose-down pitching moment, resulting in a negative moment on the airfoil.

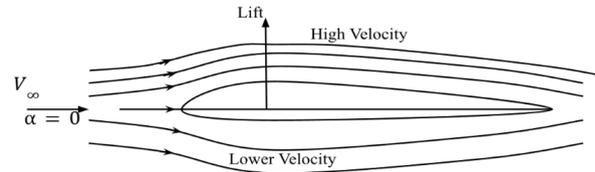


Fig. 3.3 f

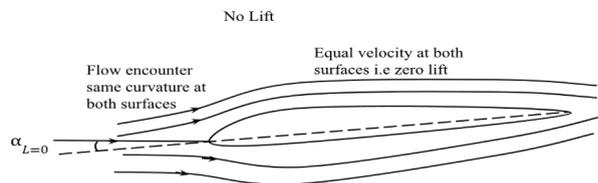


Fig. 3.3 g

→ The zero lift angle α_o or $\alpha_{L=0}$ is negative.

Note: Its magnitude is roughly equal, in degrees, to the percentage of camber.

Example: An airfoil with 2% camber, usually have

$\alpha_o = -2^\circ$

→ For symmetrical a/f it is $\alpha_o = 0$

Here $\alpha_o = \alpha_{L=0}$ = zero lift AoA (here)

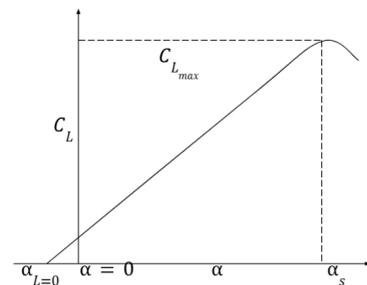


Fig. 3.3 h

Negative cambered Airfoil

→ The camber line is below the chord line.

→ Negative lift at $\alpha = 0^\circ$, So for $\alpha_{L=0} > 0^\circ$ i.e, the chord line must be positively inclined with respect to the free stream to obtain zero lift.

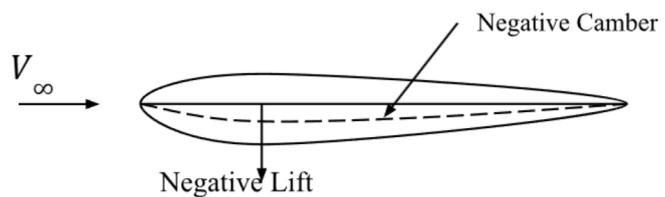
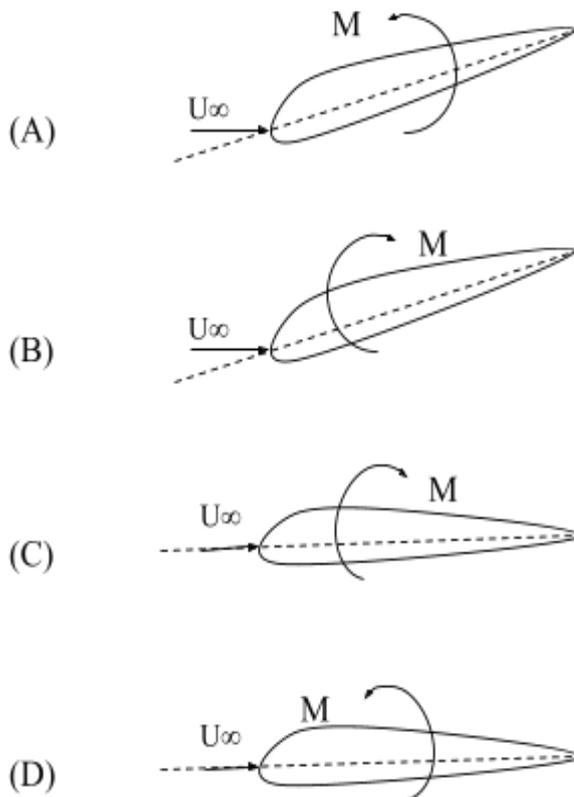


Fig. 3.3 i

→ **Question:** To achieve zero lift from a positively cambered airfoil, the chord line must be:

- A) Negatively inclined to the free stream
- B) Positively inclined to the free stream
- C) Not inclined at all to the free stream (zero angle of attack)
- D) All of the above

 *Gate 2025*
Q.19 A positively cambered airfoil is placed in a uniform flow (velocity, U_∞) at its zero-lift angle of attack. M is the corresponding pitching moment. Which one of the following representations accurately describes this scenario?



Solution: (A) For a positively cambered airfoil at the zero-lift condition, the pressure distribution around the airfoil generates a nose-down pitching moment, resulting in a negative moment on the airfoil.

For a negatively cambered airfoil it is opposite.

3.4 Pressure Distribution Around an Airfoil

When airflow passes over an airfoil, the velocity increases over the upper surface, resulting in a decrease in pressure, while the velocity on the lower surface decreases slightly—or sometimes remains nearly equal to the free-stream velocity—causing an increase in pressure.

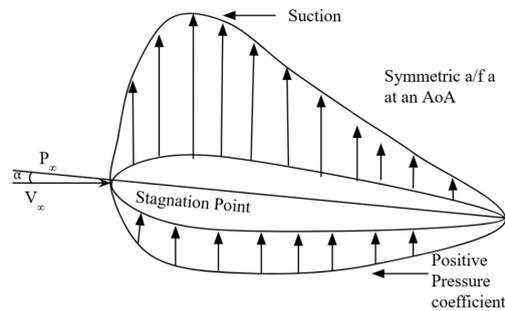


Fig. 3.4 a

By Bernoulli's theorem, we can express the pressure in terms of the pressure coefficient.

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho V_\infty^2}$$

$$C_p = 1 - \left(\frac{V}{V_\infty} \right)^2$$

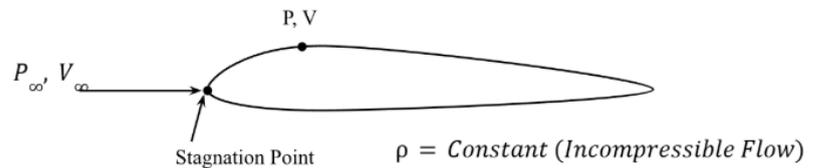


Fig. 3.4 b

- At the stagnation point $C_p = 1$ (maximum value) (upper limit), it C_p has no overall **lower limit**.
- A positive pressure coefficient, i.e., pressure greater than the free stream value. A negative C_p means a pressure less than the free stream value, i.e., referred to as “**suction pressure.**”

Note: We can observe or can note the pressure distribution around an airfoil at different AoA (angles of attack).

1. There is a large value of the lift, which is contributed mainly by the upper surface suction rather than the lower surface pressure.
2. There is little change in the lower surface pressure distribution when AoA changes from negative to positive.
3. As the AoA increases, the height of the upper surface suction peak increases, and it moves further forward. I.e., the center of pressure (CP) moves forward, i.e., towards the leading edge, with increasing AoA.
4. The zero-lift AoA is negative (for an unsymmetrical or positively cambered airfoil), and it is zero for a symmetrical airfoil. (We usually take positive camber into consideration).

Compressible Flow

Chapter 4

4.1 Gas Dynamics

Gas dynamics is the study of fluid motion when the compressibility of the fluid becomes significant. Unlike incompressible flows, gases at high speeds exhibit variations in density, pressure, and temperature. When the flow velocity approaches or exceeds the speed of sound, the behavior changes drastically, leading to phenomena such as shock waves and expansion waves. The Mach number serves as the fundamental parameter to classify flows as subsonic, transonic, supersonic, or hypersonic. Gas dynamics provides the theoretical basis for analyzing high-speed flows in applications like nozzles, diffusers, jet engines, rockets, and supersonic aircraft.

4.2 Compressibility (τ_s)

- Gas dynamics is a science that primarily deals with the behavior of gas flows in which compressibility and temperature change become significant.
- Compressibility is a phenomenon by virtue of which the flow changes its density with a change in speed.
- Compressibility may also be defined as the volume modulus of the pressure. Compressibility is given by, τ_s

i.e

$$\tau_s = -\frac{1}{v} \left(\frac{dv}{dp} \right)_s$$

$\because v = \frac{1}{\rho}$, , by differentiating, we get $dv = -\frac{1}{\rho^2} d\rho$

where v is specific volume in m^3/kg and ρ density in kg/m^3

So,

$$\tau_s = \frac{1}{\rho} \left(\frac{d\rho}{dp} \right)$$

As we know that, $\because a^2 = \frac{dp}{d\rho} \rightarrow$ Velocity of sound

$$\tau_s = \frac{1}{\rho a^2} \Rightarrow a = \sqrt{\frac{1}{\rho \tau_s}} \quad (\text{Bulk Modulus } K \text{ is given by } \because K = \frac{1}{\tau_s})$$

$$a = \sqrt{\frac{dp}{d\rho}}, \quad a = \sqrt{rRT} \quad (\text{isentropic})$$

$$a = \sqrt{RT} \quad (\text{isothermal})$$

For incompressible flow, compressibility is zero.

i.e., $\tau_s = 0$ i.e. $a = \infty$ (For incompressible Flow)

$$\because \text{Mach no. } M = \frac{V}{a} = 0$$

- So sometimes, theoretically, incompressible flow is called zero Mach number flow.
- Flows with significant compressibility are called compressible flows.
- Compressible flow is defined as variable density flow.
- It is widely accepted that compressibility can be neglected when,

$$\frac{\Delta\rho}{\rho_i} \leq 0.05$$

i.e. When $M \leq 0.03$

$$V \leq 100 \text{ m/s}$$

- We know that velocity of sound is given by,

$$a^2 = \frac{dP}{d\rho}$$

- This is Laplace's equation, and it is valid for any fluid.
- The *sound wave* is a *weak compression wave*, across which infinitesimal change in fluid properties occurs.

$$\frac{\Delta\rho}{\rho_i} = \frac{1}{2} \left(\frac{V}{a} \right)^2 = \frac{M^2}{2}$$

- This theory explains the behavior of air when compressibility and temperature changes become significant in subsonic flow.

4.2.1 Speed of Sound

The speed of sound is a critical quantity that dominates the physical properties of compressible flow.

A sound wave is propagated by molecular collision, and because the molecules of a gas are moving with an average velocity of a gas, they are moving with an average velocity of

$$\text{Average Velocity } \bar{v} = \sqrt{\frac{8RT}{\pi}}$$

&

$$\text{Velocity of Sound, } a = \frac{3}{4}\bar{v} \text{ (Kinetic Theory)}$$

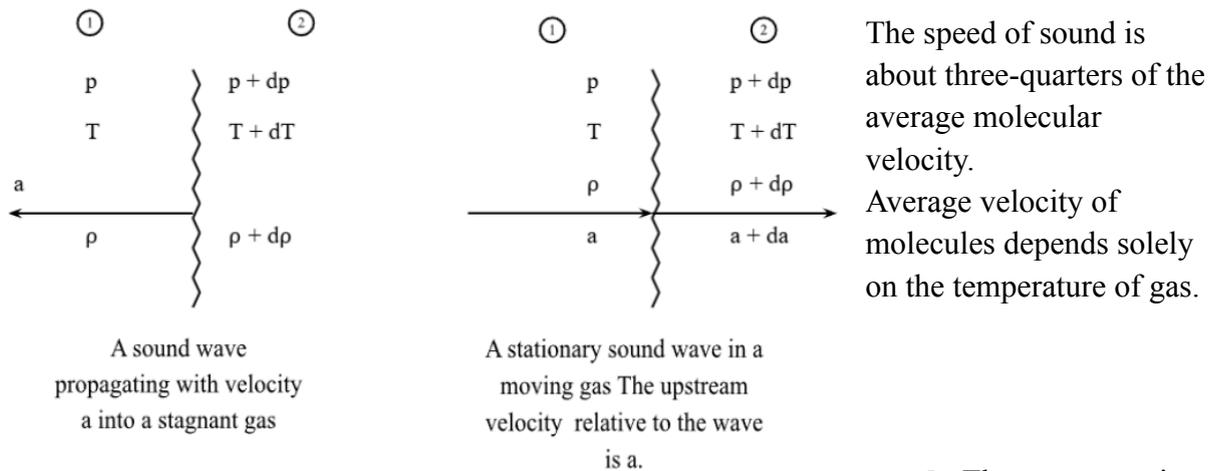


Fig. 4.2 a

and we use our macroscopic equation of continuity, momentum & energy to analyze these changes.

- In the above figure, the sound wave is nothing more than an infinitely weak, normal shock wave.
- Moreover, it is adiabatic because we have no source of heat transfer into or out of the wave.
- And gradients within the wave are tiny (the changes dp, dT, dp, and da are infinitesimal). Therefore, the influence of dissipative phenomena (viscosity & thermal conduction) is negligible. So as a result.
- The flow through the sound wave is both adiabatic and reversible, i.e., isentropic.

By applying those conservation equation

We get

$$a^2 = \frac{dp}{d\rho}$$

As discussed above, the flow through a sound wave is isentropic; hence, in the above equation, the rate of change of pressure with respect to density, $\frac{dp}{d\rho}$, is an isentropic change.

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)_s}$$

Using isentropic relation,

$$\frac{p}{\rho^\gamma} = C \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma$$

We get,

$$a = \sqrt{\frac{\gamma p}{\rho}} \quad \text{we know that the gas state equation } \frac{p}{\rho} = RT$$

$$a = \sqrt{\gamma RT}$$

$$a = \sqrt{\gamma RT} \text{ (isentropic)}$$

$$a = \sqrt{RT} \text{ (isothermal)}$$

We can conclude that the speed of sound in a calorically perfect gas is a function of temp.

→ Speed of sound: sound moves in the fluid in the form of weak finite pressure waves or pressure pulses. This velocity of waves or sound is also known as acoustic speed.

$$\text{Speed of sound, } a = \sqrt{\frac{dp}{d\rho}}$$

Across a pressure pulse the disturbance is very weak, so the flow is assumed to be adiabatic.

$$\frac{p}{\rho^\gamma} = c$$

$$\text{i. e. } \frac{dp}{d\rho} = \gamma RT$$

$$a = \sqrt{\gamma RT}$$

Note:

1. For an incompressible flow ρ is constant; therefore, $a = \infty$ the velocity of sound is ∞ .
2. For an isothermal process, the velocity of sound is given by, $a = \sqrt{RT}$. i.e, $\gamma = 1$

Compressibility (τ_s)

$$\tau_s = \frac{-1}{v} \frac{dv}{dp}$$

$$\tau_s = \frac{1}{\rho} \frac{d\rho}{dp}$$

For incompressible flow, the change in density zero, so $\tau_s = 0$

$$a \propto \frac{1}{\tau_s} \Rightarrow a = \infty$$

Mach number, $M = \frac{v}{a} = 0$ (For incompressible flow)

I.e., incompressible flow is also known as zero Mach number flow.

Note: Do not get confused about why the Mach number is said to be zero for incompressible flow. In practice, flows with Mach numbers between **0 and 0.3** are *considered* incompressible because the density changes are negligible and have little effect on the flow characteristics.

However, in a **theoretical sense**, a truly incompressible flow means that the **speed of sound is infinite**, since sound waves travel instantly through a fluid that cannot be compressed. Therefore, the **Mach number (ratio of flow velocity to speed of sound)** becomes zero for an ideal incompressible flow.

Incompressible flows are zero Mach number flows. i.e., the speed of sound is infinite for incompressible flow so the value of Mach number is zero for incompressible flow. I.e., 0 to 0.3 is the **range** in between, and flow is assumed incompressible. $M=0$ is the **value** of Mach number for Incompressible

4.2.2 Physical Meaning of Mach Number:

Consider a fluid element moving a streamline. The kinetic is internal energies per unit mass are $\frac{v^2}{2}$ & ratio of these energies.

$$\frac{v^2/2}{e} = \frac{v^2/2}{C_v T} = \frac{v^2/2}{RT/\gamma-1} = \frac{v^2}{\gamma RT} \frac{\gamma(\gamma-1)}{2} = \frac{v^2 \gamma(\gamma-1)}{2a^2}$$

$$\Rightarrow \frac{v^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2$$

We see that the square of the Mach number is proportional to the ratio of K.E. to the internal energy of the flow.

In other words, the Mach number is a measure of the directed motion of gas compared with the random thermal motion of the molecules.

Ques: Calculate the ratio of KE to internal energy at a point in an airflow where the Mach numbers are:

- $M = 2$
- $M = 20$

Sol:

- $\frac{v^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2 = 1.12$
- $\frac{v^2/2}{e} = \frac{\gamma(\gamma-1)}{2} M^2 = 112$

Aerodynamics

Note: We see that at $M = 2$, KE & internal energy are about the same. Whereas at the large hypersonic Mach number of 20, the KE is more than a hundred times larger than the internal energy.

This is the one characteristic of Hypersure flow—a high ratio of KE to e

Note: The Mach number (M) is related to the ratio of inertia force to elastic force, and since it's a ratio of forces, the Mach number is the square root of that ratio.

Mach number physically, $M = \sqrt{\frac{\text{inertia force}}{\text{elastic force}}}$

Reynold's number $Re = \frac{\text{Inertia forces}}{\text{Viscous forces}}$

(Memorize it also.)

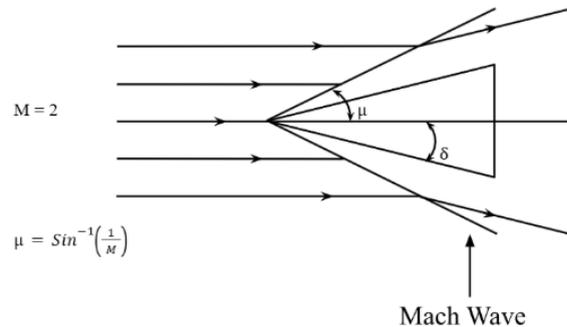


Fig. 4.2 b

 **Gate 2024**
Q.22 On Day 1, an aircraft flies with a speed of V_1 m/s at an altitude where the temperature is T_1 K. On Day 2, the same aircraft flies with a speed of $\sqrt{1.2} V_1$ m/s at an altitude where the temperature is $1.2 T_1$ K. How does the Mach number M_2 on Day 2 compare with the Mach number M_1 on Day 1?

Assume ideal gas behavior for air. Also assume the ratio of specific heats and molecular weight of air to be the same on both days.

(A) $M_2 = 0.6 M_1$

(B) $M_2 = M_1$

(C) $M_2 = \frac{1}{\sqrt{1.2}} M_1$

(D) $M_2 = \sqrt{1.2} M_1$

Solution: $M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}}$

$$M_1 = \frac{V_1}{\sqrt{\gamma RT_1}}$$

$$M_2 = \frac{\sqrt{1.2} V_1}{\sqrt{\gamma R \times 1.2 T_1}} \quad \text{as } V_2 = \sqrt{1.2} V_1 \quad \& \quad T_2 = 1.2 T_1 K$$

$$\Rightarrow M_2 = \frac{V_1}{\sqrt{\gamma RT_1}} = M_1$$

$$\Rightarrow M_2 = M_1$$

→ **Small Disturbance:** When the apex angle of wedge δ is small, the disturbances will be small. In such a case, the deviation of the streamline will be small, and there will be an infinitesimally small increase of pressure across the Mach cone.

→ **Finite Disturbance:** When wedge angle δ is finite, then disturbances are finite, and then the wave is not called a Mach wave but a shock wave with shock angle β .

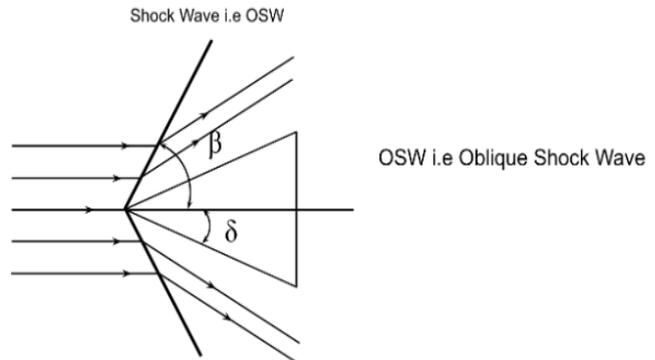


Fig. 4.2 c

Limiting Case of Oblique Shock Wave Angle:

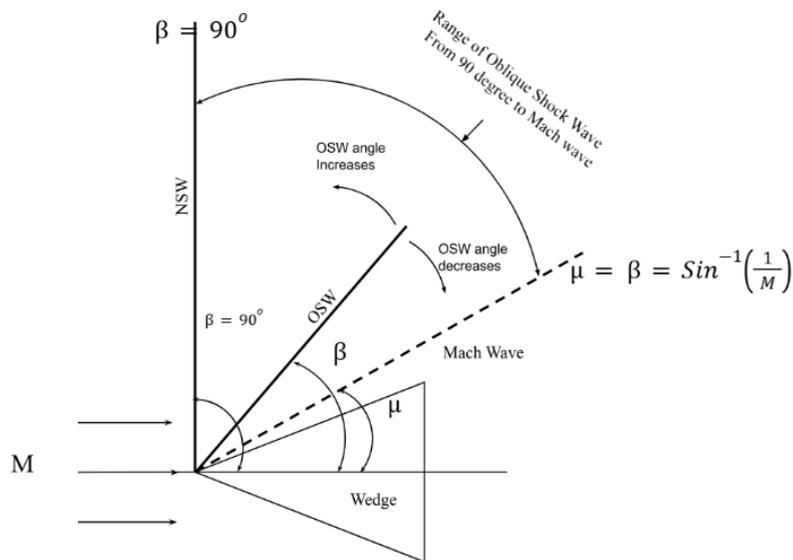


Fig. 4.2 d

$$\mu \leq \beta \leq 90^\circ$$

$$i.e \sin^{-1}\left(\frac{1}{M}\right) \leq \beta \leq 90^\circ$$

Special Topic

Chapter 5

5.1 Viscous Flow: Shock Wave / Boundary Layer Interaction

Shock waves and boundary layers normally do not mix —when they interact, serious flow problems can occur. In supersonic flows, this shock-wave/boundary-layer interaction happens often, so it is important to understand the basics.

Imagine supersonic flow over a surface where an oblique shock wave hits the surface at a point (Figure 1). In an inviscid (no viscosity) case, the incident shock strikes at point B, creating a reflected shock from the same point. This point has a sudden, large pressure increase, producing an extremely strong adverse pressure gradient.

Now, if there is a boundary layer along the wall, this huge adverse pressure gradient at point B will cause the boundary layer to separate from the surface. This is the core of the interaction: the shock wave imposes a strong adverse pressure gradient, the boundary layer separates, and then both affect each other.

Figure 2 shows this interaction more clearly. Here, a shock wave hits a boundary layer. The boundary layer grows along the plate, with subsonic flow near the wall and supersonic flow near its outer edge. When the shock wave hits, the big pressure rise forces separation. Because high pressure can move upstream through the subsonic part of the boundary layer, separation happens earlier than in the ideal inviscid case.

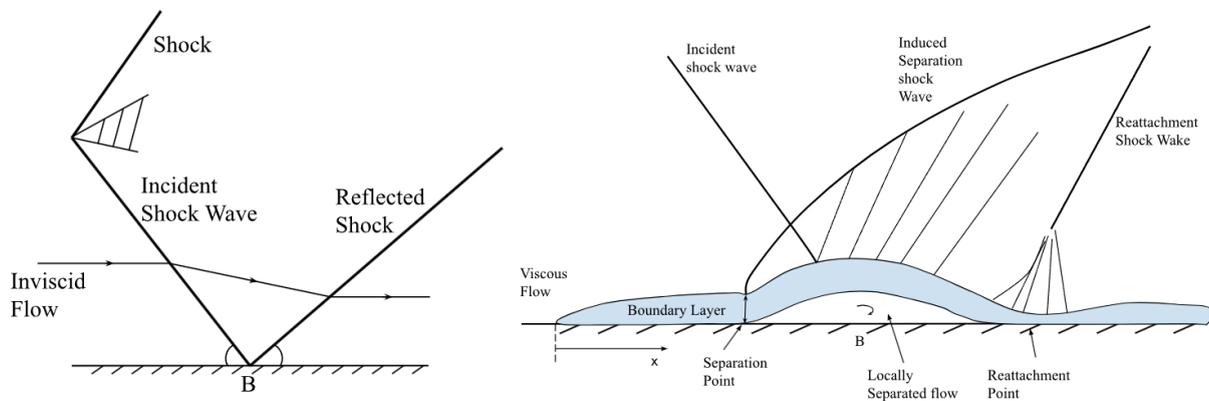


Fig. 5.1 a

The separated boundary layer bends the external supersonic flow toward itself, creating a separation shock. Further downstream, the boundary layer turns back and reattaches to the wall, causing a reattachment shock. Between these shocks, the boundary layer motion away from itself creates expansion waves.

At the reattachment point, the boundary layer is thin, the pressure is high, and local aerodynamic heating is intense. Away from the wall, the separation and reattachment shocks join to form the reflected shock expected in the inviscid case (Figure 1).

The severity of this interaction depends on the boundary layer type:

- Laminar layers separate more easily, so the effects are more severe.

→ Turbulent layers resist separation better, but the basic interaction pattern is similar.

The shock-wave/boundary-layer interaction greatly affects the pressure, shear stress, and heat-transfer patterns along the wall. One of the most important effects is the very high local heat-transfer rate at the reattachment point—at hypersonic speeds, this rate can be up to 10 times higher than at nearby locations.

For example, Figure 3 shows the wall pressure distribution in the interaction region. Here, x is the distance along the wall, measured from the theoretical impingement point of the incident shock in the inviscid case. The graph shows a step-like pressure rise with an intermediate plateau, which is a typical pattern for shock-wave/boundary-layer interaction. The external flow is Mach 3 ahead of the shock, and the boundary layer is turbulent. The solid line shows CFD results, and the circles show experimental data. Notice that the pressure increase begins about four times the boundary-layer thickness upstream of the theoretical impingement point.

Figure 4 shows the shear stress distribution. In the separated flow region, τ becomes small and even negative (reverses in direction) due to low-energy recirculating flow.

Because separated flow causes an increment in total pressure loss and high peak heat-transfer rates, engineers usually try to avoid shock-wave/boundary-layer interactions when designing supersonic aircraft and flow devices. However, avoiding them completely is difficult—they are common in real supersonic flows.

Interestingly, modern designs sometimes use the separated flow from such interactions to improve off-design performance of jet engine exhaust nozzles or for certain flow control applications. So, while often a problem, this phenomenon can also be turned into an advantage in specific cases.

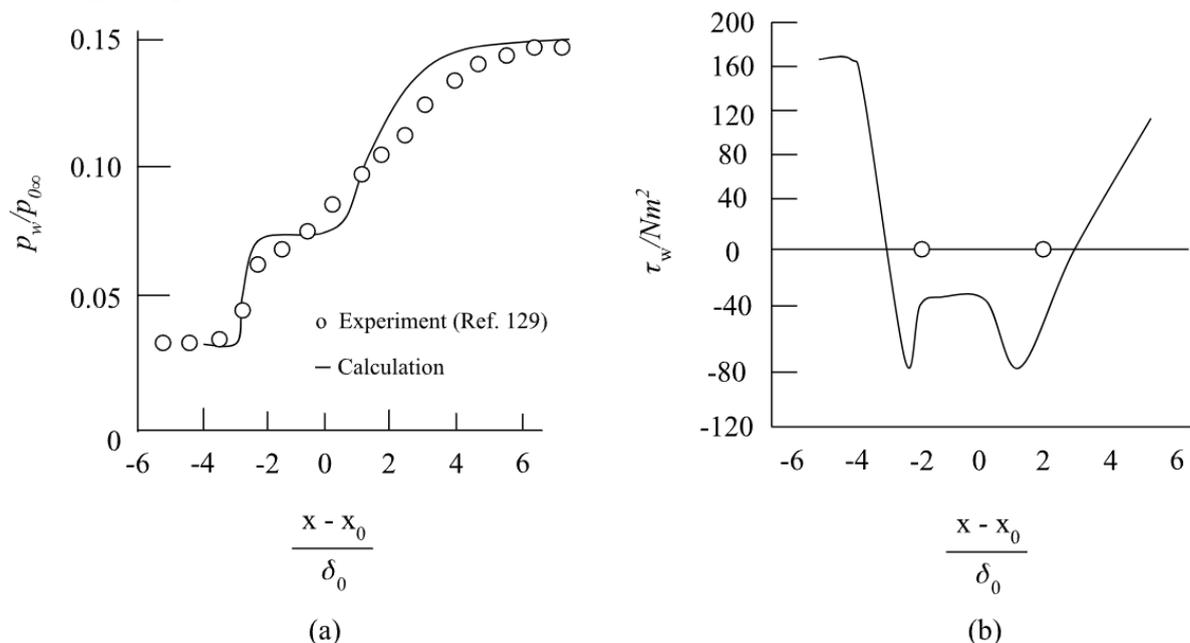


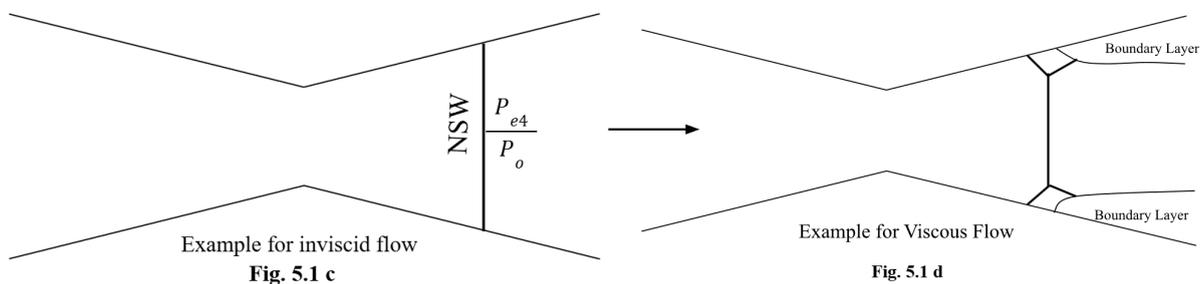
Fig. 5.1 b

5.1.1 Shock Wave / Boundary Layer Interaction Inside Nozzle

In this case, the pressure ratio $\frac{P_{e4}}{P_o}$ is such that a normal shock wave forms inside the nozzle. This is the classic inviscid flow situation.

In reality, a boundary layer grows along the nozzle wall, and the shock wave interacts with it. One possible result of this interaction is shown in Figure A. The adverse pressure gradient across the shock causes the boundary layer to separate from the nozzle wall. Near the wall, at the two “feet” of the shock, a lambda-type shock pattern forms. The main core flow of the nozzle, now separated from the wall, continues downstream through an almost constant area.

This is an example of how viscous flow effects can change the ideal inviscid flow picture.



5.2 Compressible Flow Visualization

In supersonic flows, air density changes are large enough to be photographed using optical techniques sensitive to density variations. Flow visualization methods help make the fluid motion visible either by adding tracer materials or by using optical effects caused by changes in the fluid’s properties, such as refractive index.

The most common optical methods for studying supersonic flows are interferometer, Schlieren, and shadowgraph. These techniques visualize shock waves, expansion waves, and density variations without disturbing the flow:

- **Interferometer:** Shows optical phase changes caused by density variations, giving a qualitative view of flow density.
- **Schlieren:** Shows deflection angles of light rays, highlighting regions of sharp density gradients, like faint shock waves.
- **Shadowgraph:** Sensitive to the second derivative of density, making strong shock waves visible.

These methods are **non-intrusive**, avoiding errors from adding particles or disturbing the flow. They mainly show **density patterns**, not velocity directly. Interferometry allows more precise quantitative analysis is conducted using fringe patterns, whereas Schlieren and

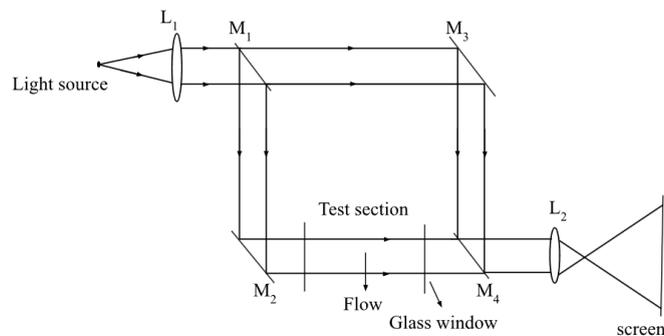
Shadowgraph techniques are simpler, cheaper, and effectively visualize flow structures at high speeds.

These techniques require high-quality optical components and precise setups to yield clear results.

5.2.1 Interferometer for Supersonic Flow Visualization

The **interferometer** is an optical method used to **qualitatively determine the density field** in high-speed flows. The most common type for gas density measurements is the **Mach-Zehnder interferometer**.

- **Principle:** Light has a wavelength and speed that change when it passes through a medium with varying density. The **refractive index** of the gas affects the light speed and is related to its density by the Gladstone-Dale equation.
- **Setup:** Light from a source is split into two beams. One passes through **room air**, the other through the **flow in the test section**. The beams recombine on a screen.



Mach-Zehnder interferometer

Fig. 5.2 a

- ◆ If paths are identical → constructive interference → **uniform bright patch**.
- ◆ If density changes in the flow → phase difference between beams → interference patterns appear as **dark and white bands** (fringes).

→ Interpretation:

- ◆ Parallel fringes far from the flow indicate **uniform density**.
- ◆ Kinks or distortions in the fringes indicate **density changes**, like **barrel shocks or Mach disks** in a supersonic jet.

→ Quantitative evaluation:

- ◆ A **phase difference of half a wavelength** results in a **black band** (destructive interference).
- ◆ A **full wavelength difference** results in a **white band** (constructive interference).
- ◆ Partial differences appear as **gray zones**.

- ◆ The resulting interferogram shows alternating black, white, and gray fringes that represent the flow density distribution.

By slightly rotating the interferometer mirrors, we obtain equally spaced vertical fringes. This initial setup makes it easier to measure the phase change per fringe and determine the flow's density field.

- In an interferogram, adjacent dark bands correspond to regions where light waves are out of phase by one wavelength.
- Let a and b be two such regions. The difference in light travel time through a and b depends on the speed of light in the medium and the length of the test section.
- The refractive index n of the gas relates the light speed in the medium to the speed of light in a vacuum:

$$n = \frac{c_{vac}}{c_{medium}}$$

- Using the Gladstone-Dale equation, the refractive index is linked to gas density, allowing us to calculate the density at each dark fringe:

$$\rho - \rho_{ref} = \frac{N_b - N_a}{LK}$$

where K is the Gladstone-Dale constant and L is the test-section length.

- This method, called “infinite-fringe” interferometry, assumes a uniform light field when there is no flow.
- Although it allows quantitative density calculation, high accuracy requires extremely precise optical components, which is a practical limitation.

5.2.1.1 Fringe-Displacement Method

The fringe-displacement method is a more accurate version of the infinite-fringe interferometer technique.

- By slightly rotating one mirror, the initially in-phase rays become out of phase, producing uniformly spaced dark and white fringes on the screen, even when there is no flow.
- If the air density in the test section changes, all wave fronts shift, causing the fringes to move perpendicular to their orientation. The amount of fringe shifting indicates the density change.

$$\rho_2 - \rho_1 = \frac{\lambda_{vac}}{LK} \frac{l}{d}$$

Important Conclusions

Chapter 6

This chapter presents a powerful collection of crucial concepts and key results that hold high importance both conceptually and exam-wise. The topics are arranged randomly, not sequentially—so study each one with full focus. Mastering these points will greatly strengthen your fundamentals and boost your confidence for the exam. It is recommended that you be at least a 3rd- or 4th year engineering student who is genuinely and seriously preparing for the GATE examination.

6.1 Lift, Drag, and Moment

$$L = \frac{1}{2} \rho V^2 S C_L$$

$$D = \frac{1}{2} \rho V^2 S C_D$$

$$M = \frac{1}{2} \rho V^2 S c C_M$$

All aerodynamics coefficients C_L , C_D & C_M are the function of α , Mach number (M) & Reynold's number (Re).

i.e

$$C_L \propto f(\alpha, M, Re)$$

$$C_D \propto f(\alpha, M, Re)$$

$$C_m \propto f(\alpha, M, Re)$$

Note: The Mach number comes into play because of the **compressibility** of the flow. If the flow is incompressible, Mach number is meaningless and absent from expressions.

Reynolds number comes into the picture because of the **viscous nature** of fluid. If the flow is inviscid, then there is no Reynolds number at all.

Case I: If the fluid is incompressible, then certain conditions apply.

$$C_L, C_D, C_M \propto f(\alpha, Re)$$

Case II: If a fluid is inviscid & incompressible, then.

$$C_L, C_D, C_M \propto f(\alpha) \text{ only}$$

In aerodynamics, inviscid & incompressible flow is known as an ideal flow.

6.2 Linear ordinary differential equation (PDE's)

Just for the time being we will look question number 3, GATE 2008

Ques: Which of the following equations is a LINEAR ordinary differential equation?

- (A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y^2 = 0$ Non-linear ($2y^2$ non-linear term)
- (B) $\frac{d^2y}{dx^2} + y\frac{dy}{dx} + 2y = 0$ Non-linear ($y\frac{dy}{dx}$ non-linear term)
- (C) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0$ Linear (no non-linear term)
- (D) $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} + y = 0$ Non-linear ($\left(\frac{dy}{dx}\right)^2$ non-linear term)

PDE should not contain the multiplication of the dependent variable or its derivatives, called a LINEAR ordinary differential equation.

Explanation:

Here the dependent variable is y , as $y = f(x)$

And derivatives of the dependent variable are $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ etc.

ODE will be linear only if it contains no such terms.

$y \times y$, $y \times \frac{dy}{dx}$, $\frac{dy}{dx} \times \frac{dy}{dx}$, $\frac{dy}{dx} \times \frac{d^2y}{dx^2}$, $\frac{d^2y}{dx^2} \times \frac{d^2y}{dx^2}$ or $\frac{d^ny}{dx^n}$, multiplication (Higher Order)

6.3 Clarification of Partial Differential Equation

The most general equation of PDE is given by,

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial xy} + C\frac{\partial^2\phi}{\partial y^2} + D\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} + F = 0$$

If,

$$B^2 - 4AC < 0 \rightarrow \text{Elliptical PDE}$$

$$B^2 - 4AC = 0 \rightarrow \text{Parabolic PDE}$$

$$B^2 - 4AC > 0 \rightarrow \text{Hyperbolic PDE}$$

$$\text{Ex: } \phi_{xx} + (1 - M^2)\phi_{yy} = 0$$

Here $A = 1$, $B = 0$

(A) If $M = 1$, Then, $C = 0$ (Sonic)

$$B^2 - 4AC = 0 - 0 = 0$$

(Parabolic)

(B) If $M < 1$, (Subsonic)

Then, $C = +ve$

$$0 - 4 \times 1 \times (+ve C) \Rightarrow -ve$$

(Elliptical)

(C) If $M > 1$, Supersonic

Then, $C = -ve$

$$0 - 4 \times 1 \times (-ve C) \Rightarrow +ve$$

(Hyperbolic)

So now it is time to look Gate 2010, Question Number 5

Ques: The linear second-order partial differential equation

$$5 \frac{\partial^2 \phi}{\partial x^2} + 3 \frac{\partial^2 \phi}{\partial xy} + 2 \frac{\partial^2 \phi}{\partial y^2} + 9 = 0 \text{ is}$$

(A) Parabolic (B) Hyperbolic (C) Elliptic (D) None of the above

Explanation: Find the value of $B^2 - 4AC$

Here $A = 5$, $B = 3$, $C = 2$

$9 - 4 \times 5 \times 2 = -ve$ i.e., elliptical PDE

 Gate 2025
Q14 The partial differential equation $\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial xy} + 2 \frac{\partial^2 \phi}{\partial y^2} = 0$ is _____.

(A) Elliptic (B) hyperbolic (C) parabolic (D) of mixed type

Solution: Find the value of $B^2 - 4AC$

Here $A=1$, $B=4$ and $C=2$

Then $B^2 - 4AC = 16 - 8 = 8$, +ve value, i.e. >0 i.e Hyperbolic

 Gate 2024
Q.12 The acceleration of a body travelling in a straight line is given by

$a = -C_1 - C_2 v^2$ where v is the velocity, and C_1, C_2 are positive constants. Starting with an initial positive velocity v_0 , the distance travelled by the body before coming to rest for the first time is:

(A) $\frac{1}{2C_2} \ln \left(1 + \frac{C_2}{C_1} v_0^2 \right)$

$$(B) \frac{1}{2c_2} \ln\left(1 - \frac{c_2}{c_1} v_0^2\right)$$

$$(C) \frac{1}{2c_2} \ln\left(c_1 + c_2 v_0^2\right)$$

$$(D) \frac{1}{2c_2} \ln\left(1 + c_2 v_0^2\right)$$

Solution: The acceleration is given by: $a = -c_1 - c_2 v^2$

where v is the instantaneous velocity, c_1 & $c_2 = \text{const}$.

$$a = \frac{dv}{dt} = -c_1 - c_2 v^2$$

We need to find the distance travelled before the body first comes to rest (i.e. until $u=0$)

$$\therefore \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx}$$

$$\Rightarrow a = v \frac{dv}{dx} = -c_1 - c_2 v^2$$

$$\Rightarrow \frac{dx}{dv} = \frac{v}{-c_1 - c_2 v^2}$$

$$\text{So displacement; } x = \int_{v_0}^x \frac{v}{-c_1 - c_2 v^2} dv$$

$$\Rightarrow x = \int_{v_0}^0 \frac{v}{-c_1 - c_2 v^2} dv$$

Solve the integral, Let $u = c_1 + c_2 v^2 \Rightarrow du = 2c_2 v dv$

$$\int \frac{v}{c_1 + c_2 v^2} dv = \frac{1}{2c_2} \int \frac{du}{u} = \frac{1}{2c_2} \ln u$$

Limits

$$x = \frac{1}{2c_2} \left[\ln(c_1 + c_2 v^2) \right]_0^{v_0}$$

$$x = \frac{1}{2c_2} \ln \left[\frac{(c_1 + c_2 v_0^2)}{c_1} \right]$$

6.4 The Knudsen number (Kn) is a dimensionless number that tells us whether a gas flow can be treated as a continuous fluid or whether the molecular nature of the gas must be considered.

It is defined as $Kn = \frac{\lambda}{L}$

where:

- λ = mean free path of the gas molecules (average distance traveled by a molecule between two successive collisions)
- L = characteristic length of the flow system (e.g., diameter of a pipe, chord length of an airfoil, gap between plates, etc.)

Flow Regimes Based on Knudsen Number:

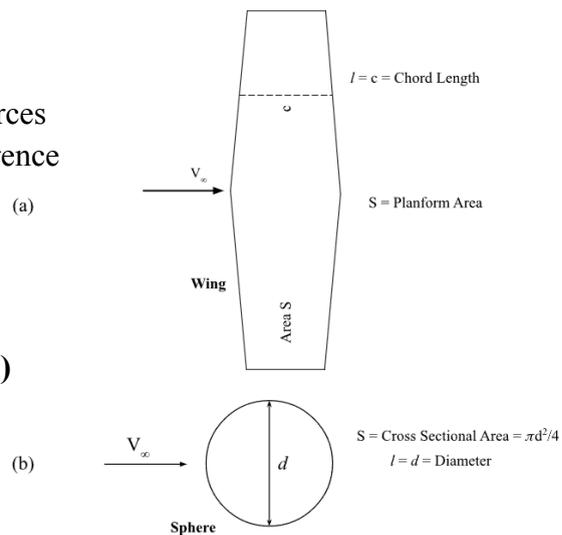
- I. $Kn < 0.01 \rightarrow$ Continuum Flow
- II. $0.01 < Kn < 0.1 \rightarrow$ Slip Flow
- III. $0.1 < Kn < 1.0 \rightarrow$ Transition Flow
- IV. $Kn > 1.0 \rightarrow$ Free Molecule Flow

6.5 Reference area

In aerodynamics, it is convenient to express forces and moments in dimensionless form using reference areas and lengths. This allows comparison of aerodynamic characteristics across different bodies and flow conditions.

The **reference area (S)** and **reference length (l)** depend on the geometry of the body.

For example, a wing uses its planform area and mean chord length, while a sphere uses its cross-sectional area and diameter.



6.6 Important Points

- Total drag on any aerodynamic body consists of pressure drag and skin friction drag, caused by pressure and shear stress on the surface.
- Pressure drag (or form drag) mainly occurs due to flow separation, while skin friction drag is caused by surface shear.

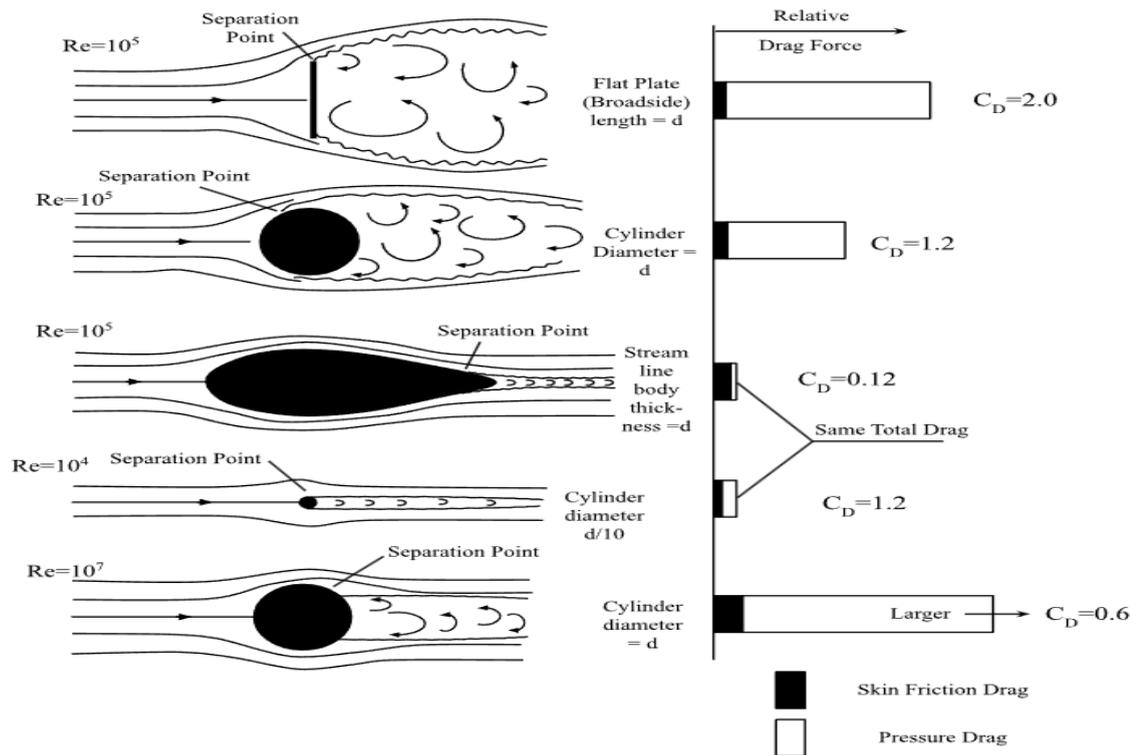
- Blunt bodies have drag dominated by pressure drag, whereas streamlined bodies have drag mainly due to skin friction.
- For a flat plate at zero angle of attack, drag is entirely due to skin friction (no pressure drag).
- The skin friction coefficient (C_f) strongly depends on the Reynolds number (Re) — it decreases as Re increases, and turbulent flow has a higher C_f than laminar flow at the same Re.
- The skin friction coefficient (C_f) for a flat plate typically ranges from 0.001 to 0.01, which is much smaller than the drag coefficients of other bodies due to the different reference areas used (planform area vs. cross-sectional area).
- A flat plate is not a practical aerodynamic shape because it has no thickness or volume; hence, real aerodynamic bodies like airfoils are considered.
- For an NACA 63-210 airfoil, the drag coefficient (C_D) at low angles of attack is about 0.0045, showing laminar flow behavior. As the angle of attack increases, turbulent transition and flow separation occur, increasing pressure drag.
- Typical airfoil drag coefficients range from 0.004 to 0.006, and most of this drag is skin friction under smooth flow conditions.
- The drag coefficient depends not only on the Reynolds number (Re) but also on the Mach number (M); as the Mach number increases, compressibility effects significantly influence drag behavior.

In reality, air is a viscous fluid, and viscosity causes deviations from ideal behavior. As a result, actual airfoils experience both lift and drag, making real fluid flow analysis essential for practical aerodynamics.

Note : Aerodynamic performance is expressed using nondimensional coefficients—lift C_L , drag (C_D), and moment (C_M)—which depend on body shape and Reynolds number (Re).

Typical drag coefficients range from $C_D \approx 2.0$ for a flat plate broadside to $C_D \approx 1.2$ for a circular cylinder, to $C_D \approx 0.12$ for a streamlined body, illustrating the importance of streamlining in reducing flow separation and wake formation. Drag on any body is composed of pressure drag (form drag) due to separation and wake, and skin friction drag due to surface

shear. Blunt bodies, such as cylinders and flat plates, are dominated by pressure drag, while streamlined bodies are dominated by skin friction. The drag coefficient due to skin friction (C_f) decreases with increasing Re and depends on flow type, being higher for turbulent than laminar flow. In practical applications, Re is often in the millions, and understanding the relative contributions of pressure and skin friction drag is crucial for aerodynamic design, realistic performance prediction, and distinguishing between blunt and streamlined configurations.



Blunt body \Rightarrow a body where most of the drag is pressure drag

Streamlined body \Rightarrow a body where most of the drag is skin friction drag

6.7 Airfoil in an Ideal Fluid

In an ideal (inviscid) fluid, the airflow around a symmetric airfoil can be analyzed using Bernoulli's and continuity equations. When the airfoil is aligned with the free stream, the flow slows and stagnates at the nose, resulting in maximum pressure. As the flow moves over the curved surface, velocity increases and pressure decreases, reaching the highest velocity and lowest pressure at the thickest point. Beyond this point, velocity decreases and pressure rises again toward the trailing edge.

On the front portion of the airfoil, there is a negative pressure gradient (pressure decreases along the surface), while on the rear portion, there is a positive pressure gradient (pressure increases).

About the Author

Krishna Parmar is a highly accomplished Aerospace Engineer, academician, and author, with extensive experience in both industry and academia. He earned his **Master of Technology in Aerospace Engineering** from **IIT Madras, India**, and is an **Associate Member of the Aeronautical Society of India**, New Delhi.

Krishna began his academic journey with a **Bachelor of Science from the University of Rajasthan**, followed by a **Bachelor of Technology from the Aeronautical Society of India**. He further enhanced his expertise with postgraduate degrees in **Management from Sir Sandford Fleming College, Peterborough, Canada**, and **St. Lawrence College, Toronto, Canada**.

In addition to his formal education, Krishna has **completed certification in Non-Destructive Testing in Aerospace Engineering**, earning the **Roll of Honour** for outstanding performance. This achievement underscores his technical excellence and dedication to advancing safety and quality in aerospace engineering.

Krishna has served as a **Professor in the Department of Aerospace Engineering**, where he shared his knowledge and experience with aspiring engineers. He also conducts **GATE coaching classes**, guiding students to succeed in competitive examinations. Beyond academia, Krishna is a **content creator and author**, producing high-quality educational material to reach and inspire a wider audience.

Originally from **Bharatpur, Rajasthan, India**, Krishna currently resides in **Toronto, Canada**. His diverse academic background, professional experience, and commitment to aerospace education make him a respected figure in the aerospace engineering community and a valuable resource for students and professionals alike.

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Navin Soni

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Navin Soni is a globally experienced aerospace professional currently leading advanced engineering initiatives at Jaguar Land Rover in the UK. With a solid academic foundation—M.Tech in Aerospace Engineering from IIT Madras and a B.Tech from AeSI—he brings deep technical expertise and innovative thinking to every role. His career spans GE Aviation (India & USA), Siemens (India & Sweden), and VinFast (Vietnam), where he delivered impactful solutions in aircraft structures and engineering systems. At Jaguar Land Rover, he applies his aerospace background to shape the future of automotive design and performance. Navin combines elite education with real-world engineering across continents—making him a standout leader in global technology and innovation.

Navin Soni



Anil Sharma

M.Tech IIT Madras | Aerospace Expert | Civil Administrator |
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Anil Sharma, an aerospace engineering professional with an M.Tech from IIT Madras and B.Tech from AeSI, brings a unique blend of technical excellence and public service. Currently serving as a Civil Administrator in the border state of Rajasthan (India), Anil combines his deep knowledge of aerospace engineering with leadership in governance. He began his career as a CFD Engineer at VJCS Pvt. Ltd. (Pune), where he applied computational fluid dynamics to solve complex engineering problems. A proud achiever of the prestigious AFCAT exam, Anil is also passionate about teaching and mentoring. As a technocrat and dedicated educator, he guides GATE Aerospace aspirants, helping them secure top ranks with his expert insights and structured mentoring approach.

Anil Sharma



Ramprakash Banjare

M.Tech IIT Madras | Aerospace Engineer | Composite Structures Expert at GE Aerospace

Ramprakash Banjare is a seasoned aerospace engineer with over 12 years of experience in the design, analysis, and certification of advanced composite structures. An alumnus of IIT Madras (M.Tech, Aerospace Engineering), he brings deep technical expertise and a track record of innovation. Currently at GE Aerospace, Ramprakash has led critical projects across aerospace and renewable energy, including GE9X engine components and next-generation offshore wind turbine blades. He is highly proficient in FEA tools like ANSYS, ABAQUS, and HyperMesh, and excels in composite manufacturing and structural optimization. Known for his engineering impact and cross-functional leadership, he has received multiple awards for excellence in innovation and collaboration.

Ramprakash Banjare



Vijit Khatri

Vijit Khatri – Lead Engineer at John Deere | Thermal & Aerospace
Engineering Expert

Vijit Khatri is a skilled engineering professional currently serving as a Lead Engineer at John Deere India Pvt. Ltd., where he specializes in mechanical and thermal systems for advanced agricultural and automotive applications. He holds an M.Tech in Mechanical Engineering (Thermal) from NIT Surat and a B.Tech in Aerospace Engineering from the Aeronautical Society of India (AeSI)—demonstrating a strong academic foundation across both thermal and aerospace disciplines. At John Deere, Vijit plays a key role in the development and optimization of engine cooling, HVAC, and heat transfer systems, contributing to the advancement of high-efficiency machinery platforms. His work bridges core mechanical principles with real-world engineering needs, ensuring performance, durability, and sustainability. He brings deep experience in thermal-fluid systems, component testing, and simulation tools, and has been instrumental in resolving complex technical challenges through structured problem-solving and innovation. Known for his methodical approach, cross-functional collaboration, and commitment to continuous improvement, Vijit is passionate about leveraging engineering to create smart, future-ready solutions that support global agricultural and industrial progress.

Vijit Khatri



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Expert in Aircraft Structural Integrity

Naeem Khan is a highly accomplished aerospace engineer and NDT (Non-Destructive Testing) expert with over 12 years of hands-on experience ensuring the structural safety and performance of aircraft. With a strong academic foundation and an M.Tech in Aerospace Engineering, Naeem brings a rare blend of technical excellence, field expertise, and leadership in aviation maintenance and inspection. Renowned for his precision and problem-solving skills, Naeem has played a pivotal role in the inspection and lifecycle management of airframe structures, advanced composites, and critical aerospace components. His proficiency spans all major NDT techniques—including Ultrasonic Testing (UT), Eddy Current Testing (ET), Radiographic Testing (RT), Magnetic Particle Testing (MT), and Dye Penetrant Testing (PT). Naeem has led multidisciplinary inspection teams, contributed to major civil and defense aviation projects, and delivered measurable impact in safety assurance and regulatory compliance. He is also known for mentoring upcoming professionals, driving quality excellence, and implementing cutting-edge inspection technologies. Whether optimizing inspection protocols or troubleshooting complex structural anomalies, Naeem is committed to engineering excellence and elevating aviation standards.

Naeem Khan



Deepak Kumar Sharma

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Deepak Kumar Sharma is a highly skilled Aero-Thermal Engineer with 10+ years of experience in electric vehicle thermal systems, aerodynamics, and energy storage solutions. He holds a Master's in Renewable Energy from IIT Madras, one of India's premier institutes, and a B.Tech in Aerospace Engineering from AeSI. At Pravaig Dynamics, he leads the design of integrated thermal management systems, battery cooling, HVAC optimization, and aerodynamic enhancements—delivering cutting-edge solutions that drive both efficiency and sustainability in next-gen EVs. His core expertise spans CFD, advanced thermal simulations, and full system integration, making him a critical asset in high-impact engineering environments. Deepak is known for translating complex thermal challenges into practical, high-performance innovations. Previously at QuEST Global, he contributed to Rolls-Royce gas turbine engine development, optimizing secondary air systems and conducting detailed Aero-Thermal analyses for enhanced engine reliability and efficiency. Recognized for his cross-functional leadership, precision engineering, and forward-thinking mindset, Deepak Kumar Sharma stands at the forefront of sustainable mobility, driving progress through innovation, collaboration, and technical excellence.

Deepak Kumar Sharma



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Dr. Nazim Khan-Ph.D. in Aerospace Engineering from IIT Kanpur, an M.Tech from IIT Madras

Dr. Nazim Khan is a distinguished researcher specializing in multiscale modeling of Integrated Thermal Protection Systems (ITPS) for reusable launch vehicles (RLVs), with a focus on laser ablation of advanced ceramic coatings. He earned his Ph.D. in Aerospace Engineering from IIT Kanpur, an M.Tech from IIT Madras, and a bachelor's degree from the Aeronautical Society of India. Prior to his doctoral studies, Dr. Khan gained valuable industry experience in the automobile and aerospace sectors, contributing to advanced engineering projects. Currently, he is a Postdoctoral Fellow at IIT Kanpur, leading sponsored research initiatives on laser ablation of ceramic coatings. His work has been published in leading international journals, including AIAA Journal and Aerospace Science and Technology, and presented at prominent national and international conferences. In addition to his academic contributions, Dr. Khan advises a space-tech startup, providing strategic guidance in the development of innovative aerospace prototypes, bridging the gap between research and industry.

Nazim Khan



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Krishna Parmar is a passionate aerospace engineer and educator with a strong foundation in both academic excellence and practical industry expertise. He holds an M.Tech in Aerospace Engineering from IIT Madras and a B.Tech from the Aeronautical Society of India (AeSI), where he is also an Associate Member. Beginning his career as an Assistant Professor at PVP Siddhartha Institute of Technology, Andhra Pradesh, Krishna dedicated over six years to shaping future engineers while building a deep understanding of aerodynamics, propulsion, and aircraft structures. In 2012, he founded the IITians Aerospace Team, which later evolved into IITians Gate Academy, a premier online platform guiding GATE Aerospace aspirants with top-tier instruction from IIT and IISc graduates. Expanding his global expertise, Krishna pursued Project Management at Sir Sandford Fleming College, Canada, followed by a specialization in Non-Destructive Testing (NDT) in Aerospace Engineering. Today, he works at the intersection of aerospace inspection and education—empowering students and professionals alike through innovation, mentorship, and technical excellence.

Krishna Parmar



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